Cornell

ME 4730/5730

Assembled by Andy Ruina in Sept 2015.

Old Exam problems 2012-2014

No calculators, books or notes allowed.

The text on this page below, or something similar, will be on the covers of future exams. Please read it slowly before any exams and ask questions about anything unclear before the exam.

How to get the highest score?

Please do these things:

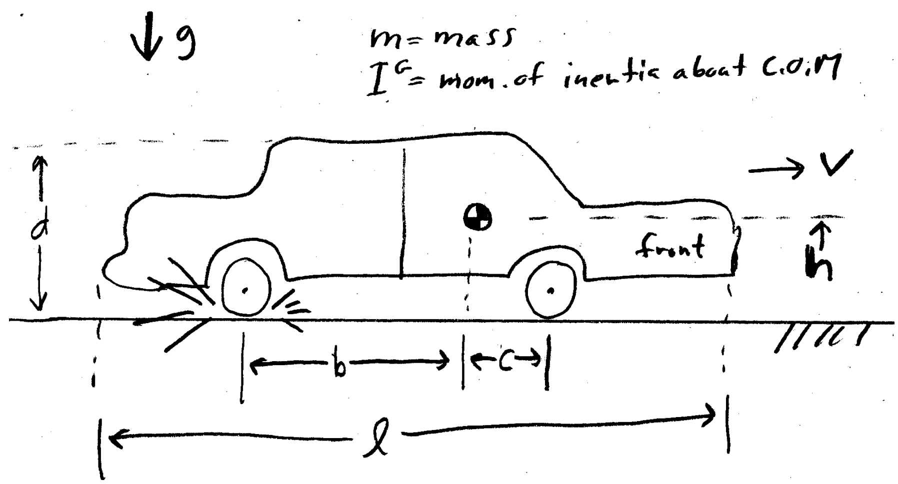
- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- ☐ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 2\pi$" instead of, say, "`T(7) = 2*pi;`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60 – 100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Fall 2012, Prelim 1, Problem 1.
0) Read the directions on the front cover.

1) 2D in side view. A car, moving to the right in the figure below, screeches to a stop, skidding the rear wheels (coefficient of friction $=\mu$, friction angle = ϕ, with $\tan\phi = \mu$). The brakes are not applied to the light front wheels which roll easily.

What is the vertical force from the ground on the rear wheels ("the" force is the sum from both wheels)?

Answer in terms of some or all of the variables on this page. Extra credit for showing that your answer agrees with one or more special cases that you can evaluate more simply.



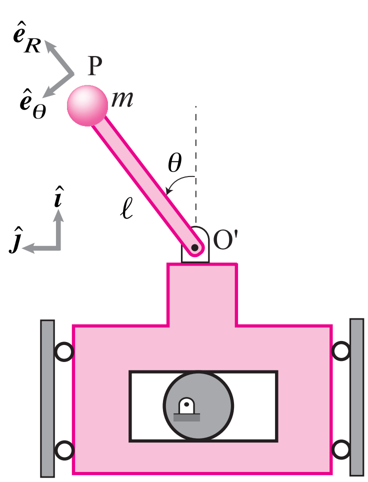
Fall 2012, Prelim 1, Problem 2.
2) A motor, not shown, is mounted a distance d off center (eccentrically) in the disk that is in a rectangular hole. Therefore the base moves up and down a distance

$$x_{0'} = d\sin\omega_0 t.$$

The rod has negligible mass. There is gravity pointing down. At $t = 0$ the rod is at θ_0 with angular speed $\dot{\theta}_0$. You are given m, g, ℓ, d, ω_0 and t_{final}.

(1) Find the equations of motion.

(2) Write Matlab code that would give θ att_{final}. Assume any non-zero values that please you for all constants that you need. The code need *not* have detailed comments. And, if you need to comment, you can use normal writing and arrows and such (you do not have to use `% standard Matlab comment style`).



Fall 2012, Prelim 1, Problem 2.

3) Assume you want to find the motion of the 4-bar linkage shown.

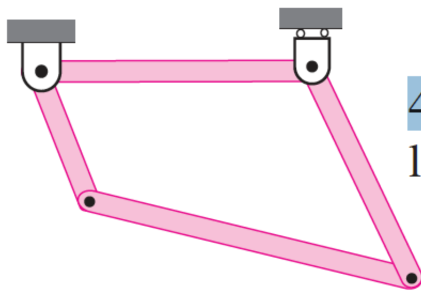
(1) List all constant parameters needs (e.g., g, etc.)

(2) How many degrees of freedom does the system have?

(3) Give an example of a good set of minimal coordinates?

(4) The big matrix for the DAEs will have how many rows and how many columns? Please be clear about the meanings of the rows and columns.

(5) Give an example of each type of equation used in a row or column (hint: there are 3 distinct types). It should be explicit (a readable equation) but need not be in matrix form (need not have all the zeros needed in the matrix). But it should be separated into appropriate left and right hand sides. Explain clearly where each goes in the matrix in terms of the vector that the matrix multiplies.



Fall 2013, Prelim 1, Problem 3.

1) A particle m moves near a fixed central mass M to which it is *attracted* with a force with magnitude $|\vec{F}_g| = GmM/r^2$. A quadratic drag also acts with magnitude $|\vec{F}_d| = cv^2$.

a) Write a vector expression for the total force $\vec{F}$ acting on the particle in terms of its position $\vec{r}$ and velocity $\vec{v}$.

b) The particle is launched from the the x axis at $x = x_0$ with a velocity $\dot{y}_0$ in the y direction.
Write the Matlab code to find the distance from the origin at given time t_{end}.
Some lines of working code are given on the next page, fill in the missing lines.
[In the 'official' solution, which is readable and not minimal, this takes 12 lines total.
It can be done with 5 impenetrable lines.].

c) For what values of the parameters G and c is linear momentum conserved? Angular momentum? Total energy?

```
%%%%%%%%%%%%%%%%%%%%%%%%
%All of this goes in one text file.
%The lines already written need no fixing
%Find the motion of a particle
function  orbit
p.c=0.1;  p.m =1; p.M = 1; p.G = 1;
x_0= 1;  ydot0 = 4;
t0= 0; tend=10;

  ---FILL IN MULTIPLE LINES HERE---

disp(['The distance is' num2str(d)]);
end

  ---FILL IN MULTIPLE LINES HERE---

end
%%%%%%%%%%%%%%%%%%%%%%%%
```

Fall 2013, Prelim 1, Problem 2.

2) A mass m hangs a variable distance $x(t)$ down from the ceiling from a spring (k, ℓ_0) and dashpot c, pulled by gravity. It is forced by a downwards force $F = F_0 \sin(\omega t)$ that is transmitted by another spring and dashpot (k, ℓ_0, c) that are connected by a bar with negligible mass $(m_1 = 0)$. You can neglect the physical dimensions of the masses.

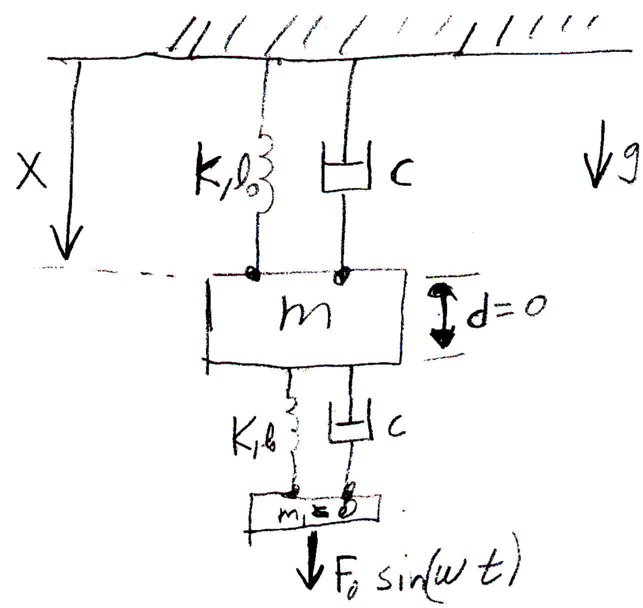
a) Find the equations of motion for the big block (ie, the ODE governing $x(t)$). That is, find a formula for $\ddot{x}$ given $k, \ell_0 c, g, m, F_0, \omega, x, \dot{x}$ and t.

b) For given initial conditions $(x_0, \dot{x}_0)$, x can be thought of as the sum of three terms: A constant term, a transient term, and a steady state term.

- **i)** Find the constant term.
- **ii)** What is the general form of the transient term, and what simpler equation does it satisfy?
- **iii)** What is the general form of the steady state term, and what simpler equation does it satisfy?

c) Assume the same system is released from rest with $F_0 = 0$. For what value of c does the mass most rapidly return to it's stationary equilibrium (holding all other variables constant)?

d) Holding all other values constant, for what value of ω is the steady state vibration amplitude of the mass the largest? If you don't know the exact value, give an approximate value (answers in terms of all variables, but for ω).



Fall 2013, Prelim 1, Problem 3.

3) Five equal masses m are in a line held apart and between two walls with 6 equal springs k. Assume the springs are relaxed in the equilibrium position. Measure the x_i relative to the equilibrium position.

a) Write out the matrices M and K so that $M\ddot{\vec{x}} + K\vec{x} = \vec{\mathbf{0}}$.

b) One normal mode is $[1\ \ -1\quad 0\quad 1\ \ -1]'$. What is its frequency of oscillation? (use what ever definition of frequency you like and define).

c) Find another normal mode and give its frequency.

d) Find still another and give its frequency.

e) Write a matlab command(s) to find the fifth (as sequenced by Matlab) normal mode and frequency. Just one mode and associated frequency are desired (Why just one? So you can show that you understand the Matlab output).

f) Of the five normal modes, what, *approximately*, is the mode shape associated with the lowest frequency (just make a guess, describe it, and justify it).

Fall 2014, Prelim 1, Problem 1.

1) Read and bead.

a) Read the cover page carefully.

b) A bead m slides frictionless on a rigid wire with the shape $y = c\sin(dx)$. Gravity g points in the $-\hat{\jmath}$ direction. Write code representing the equations of motion. Use any method that you well-explain.

Justify your work on this page. Fill in the Matlab 'Right-hand-side' m-file on the next page.

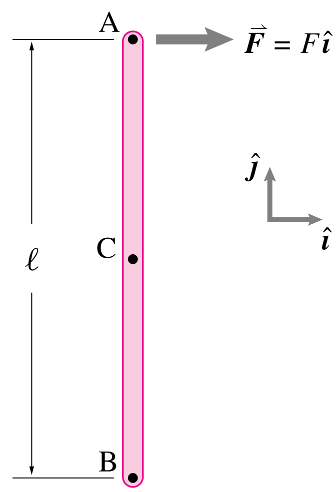
If you want to generate code using symbolic commands outside the rhs file, that is ok. No need to write out the print statements, just be clear what output would go where in the rhs file.

```
function zdot = rhs(t,z, m,g,c,d)
x = z(1);    xdot = z(2);
%fill in well commented Matlab commands below
```

```
zdot = [xdot xddot]';
end % end of function zdot
```

Fall 2014, Prelim 1, Problem 2.

2) A uniform stick with mass m and length ℓ is initially stationary when the force F is suddenly applied. Various points on the bar have various accelerations in the instant just after force application. What additional force (in addition to $F\hat{\boldsymbol{\imath}}$ at A) has to simultaneously be applied to point B to make, just after force application, the acceleration of point A $= \vec{\boldsymbol{a}}_{\mathrm{A}} = (F/m)\hat{\boldsymbol{\imath}}$?



Fall 2014, Prelim 1, Problem 3. **3)** Konig's Theorem etc.

a) Using the clearest possible reasoning, show that the kinetic energy of a system of particles can be written as a sum of two terms: E_G, associated with the motion of the center of mass, and $E_{/G}$, associated with motion relative to the center of mass.

b) Using a system of two particles find an example, or prove that none exists, where $E_G = 0$, $E_{/G} \neq 0$ and the angular momentum, relative to the center of mass, is zero: $\vec{\boldsymbol{H}}_{/G} = \vec{\boldsymbol{0}}$.

c) Consider a system of three particles floating in space without gravity, but connected by non-zero springs and non-zero dashpots. Which of these system quantities is generally (for all initial conditions and all possible combinations of springs and dashpots) constant in time and which not (explain)? For each that is not generally conserved, name special situation(s) (special values for initial conditions, spring constants or dashpot constants) where it would be conserved.

Each answer should be of the form:
"Generally conserved because ...." or
"Generally not conserved because ...
Would be conserved if ..." (can give more the one special case).

i. linear momentum

ii. Kinetic energy

iii. Total mechanical energy ($E_k + E_p$)

iv. angular momentum relative to center of mass, relative to fixed point O

v. angular momentum of center of mass, relative to fixed point O

vi. total angular momentum, relative to fixed point O

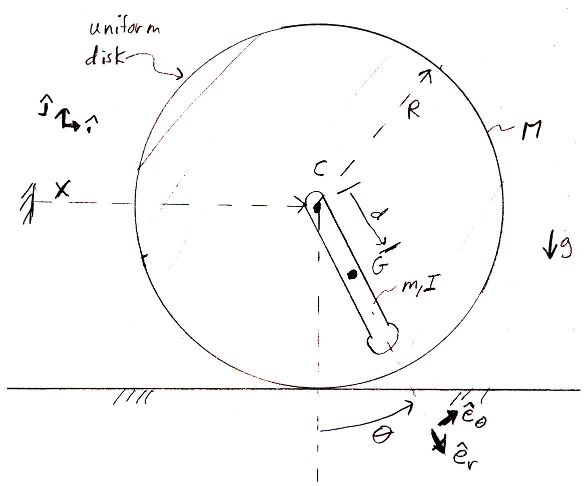
Fall 2012, Prelim 2, Problem 1.

1) 2D. A uniform disk (mass M, radius r) rolls without slip on level ground. Hanging from it's center C is a pendulum (with mass m, moment of inertia I about it's COM, and distance d from C to the pendulum's COM at G). Answer all questions in terms of $x, \dot{x}, \theta, \dot{\theta}, \hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, M, m, r, d$ and I, or an appropriate simplified subset of these.

(1) Write 2 scalar equations from which one could solve for $\ddot{x}$ and $\ddot{\theta}$.

(2) For small motions near $x = 0$ and $\theta = 0$ write the equations in standard vibration form, finding the components of the matrices M and K.

(3) By *any* means find just one normal mode: give the value of ω, the components of $\vec{v}$, and describe the mode in words.



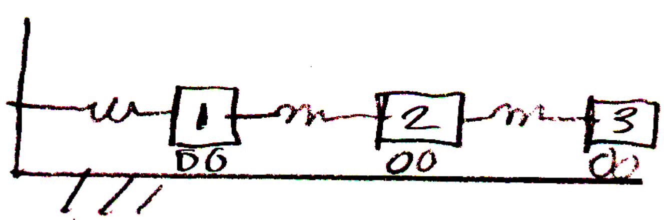
Fall 2012, Prelim 2, Problem 2.

2) Consider the one-D arrangement of three unequal masses and three unequal springs shown. Write Matlab code that would

plot the deflection (from equilibrium) of the first mass as a function of time.

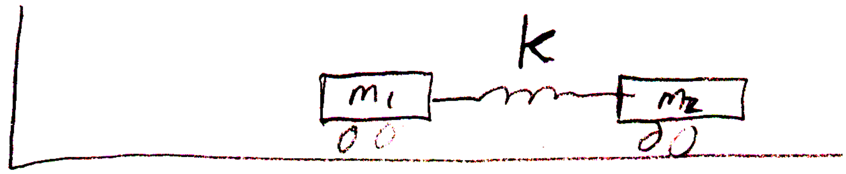
Pick non-trivial numerical values for all variables (that is, do not pick variables that especially simplify the problem).

- The initial deflections of the masses are given as $\vec{x}_0 = [111]'$.
- The initial velocities are zero.
- Use techniques from vibrations (i.e., not ode23 or Euler's method).
- As much as possible, have Matlab do the calculations (i.e., don't try to find normal modes by hand calculations or intuition).
- You can assume that none of the normal modes have $\omega = 0$.



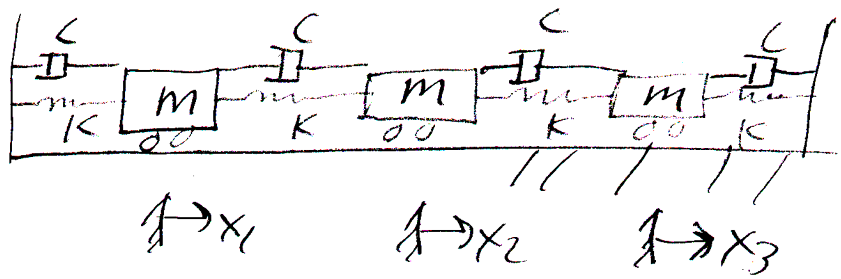
Fall 2012, Prelim 2, Problem 3.

3) 1D. Two *unequal* masses are connected to each other, and nothing else, by one linear spring. Find and describe as many normal modes as you can. That is, clearly give the mode shapes and frequencies in terms of m_1, m_2 and k. As always, clearly justify your results.



Fall 2013, Prelim 2, Problem 4 (first problem in prelim is numbered 4).

4) Three equal masses m are held apart between two walls by four equal linear springs k and four equal linear dashpots c.



a) Assume $m = 1, k = 2$ and $c = 0.01$ in some consistent unit system. Use initial conditions

$$\vec{x}(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \dot{\vec{x}}(0) = \vec{v}(0) = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

Write Matlab commands that would find $x_1(t = 10)$. You may *not* use numerical integration (e.g., *no* Euler's method, *no* ODE45, etc).

b) Can the general motion (that is, the solution for an arbitrary initial condition) of this system be found as the sum of solutions each of which is a 'mode' behaving as an independent damped oscillator? If so, describe precisely how to find these modes. If not, explain why not.

c) Assume a force $F = 3\sin(2t)$ is applied to just mass 1. In steady state, approximately what are the amplitudes of vibration of the three masses? No detailed arithmetic is desired, rather say something like 'mass 7 moves much more than mass 8 and much less than mass 4', with appropriate substitutions for 7, 8, 4 the words 'much more' and 'much less'. Use words to justify your answer.

Fall 2013, Prelim 2, Problem 5 (first problem in prelim is numbered 4).

5) Three equal point masses m move in 2D and are attracted to each other by inverse square gravity $F_{ij} = Gm_i m_j / r_{ij}^2$. There are no other forces. One possible motion is that they all travel in circles about the origin, each with the same constant $\dot{\theta} = \omega$, with the three masses on the vertices of a (rotating) equilateral triangle with sides = ℓ. Find the rate of rotation ω in terms of G, m and ℓ.

Fall 2013, Prelim 2, Problem 6 (first problem in prelim is numbered 4)

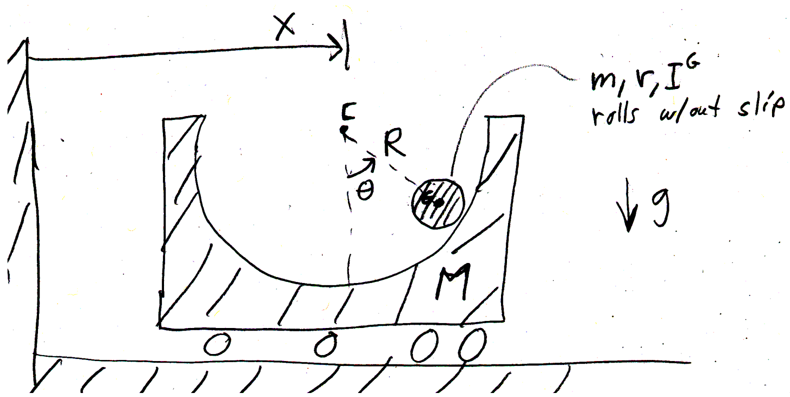
6) A uniform stick with mass m and length ℓ hangs a stationary hinge at one end. Gravity g acts. The angle of the stick from vertically down is $\theta(t)$, measured CCW.

a) Find $\ddot{\theta}$ in terms of some or all of m, g, ℓ, θ and $\dot{\theta}$ as many different ways as you can. If you have well-defined equations from which $\ddot{\theta}$ could be found, you need not do the algebra.

b) Find, using any single method of your choice, the force acting on the hinge in terms of some or all of $m, g, \ell, \theta, \dot{\theta}$ and any unit vectors you clearly define (your choice of unit vectors).

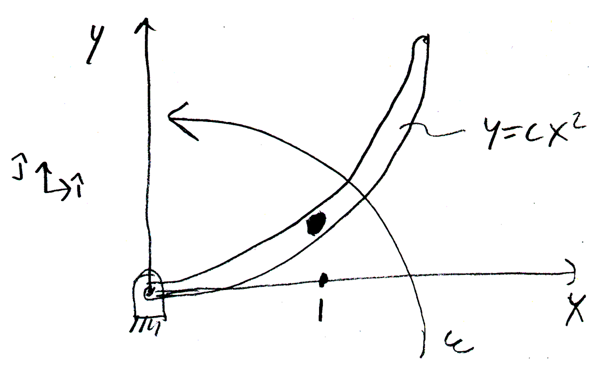
Fall 2014, Prelim 2, Problem 1.

1) Find, by whatever means pleases you (but not allowing computer algebra), the equations of motion of the system below. You need not invert the mass matrix (that is, you need not solve for $\ddot{\theta}$ and $\ddot{x}$). That is, write a clear set of equations from which you could find $\ddot{\theta}$ and $\ddot{x}$ if given $m, M, r, R, I^G, g, \theta, \dot{\theta}, x$ and $\dot{x}$. R is the distance from the center of the hollow cylinder to the center of the rolling cylinder.



Fall 2014, Prelim 2, Problem 2.

2) A rigid tube has the shape $y' = cx'^2$. It is rotated about the origin at constant ω along with it's $x' - y'$ coordinate system. Inside the tube a particle moves, due to magical forces, with constant $\dot{x}' = v_o$. At the instant of interest the $x' - y'$ axis is coincident with the $x - y$ axis and the bead is at $x' = 1$ (in some consistent units). Find the acceleration $\vec{a}$ of the bead in terms of $v_0, c, \omega, \hat{\imath}$ and $\hat{\jmath}$.



Fall 2014, Prelim 2, Problem 3.

3) Consider the equation for the undamped sinusoidally forced harmonic oscillator:

$$m\ddot{x} + kx = F_o \sin(\omega t)$$

a) Assuming $\omega \neq \sqrt{k/m}$ find a particular solution $x_p(t)$ to the governing equation.

b) As accurately as you can, plot the amplitude and phase of that solution (noting any key points on the axes).

c) Assume $x(0) = 1$ and $\dot{x}(0) = 0$, find $x(t)$.

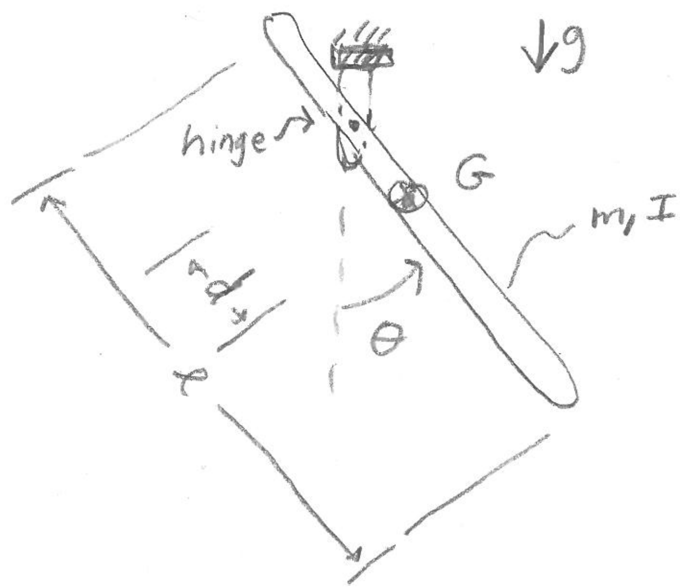
Fall 2012, Final exam, Problem 7a (first problem on exam is 7)

7a) 2D. A stick with length ℓ, mass m and moment of inertia I about its center of mass G is suspended from a hinge on the stick a distance $d < \ell/2$ from G.

(i) Find the equations of motion.

(ii) For given $\theta, \dot{\theta}, m, I, \ell$ and d find the force of the hinge on the pendulum (using any base vectors you like).

(iii) For given ℓ, m, I and gravitational acceleration g for what d is the period of small oscillation minimized. If you can reduce this to finding the root of a polynomial or transcendental equation, that is good enough, you need not find the root. [Hints: Find the equations of motion → Solve them → Find the period of small oscillations → Minimize.]



Fall 2012, Final Exam, Problem 7b.

7b) A block with mass M slides without friction on a flat level surface. The top surface has slope γ. A smaller block with mass m slides without friction on the sloped top of the lower block.

(i) Find two scalar equations from which you could, if you liked, solve for $\ddot{s}$ and $\ddot{x}$ in terms of some or all of $x, \dot{x}, s, \dot{s}, \gamma, m, M$ and g [that is, you need not invert the mass matrix].

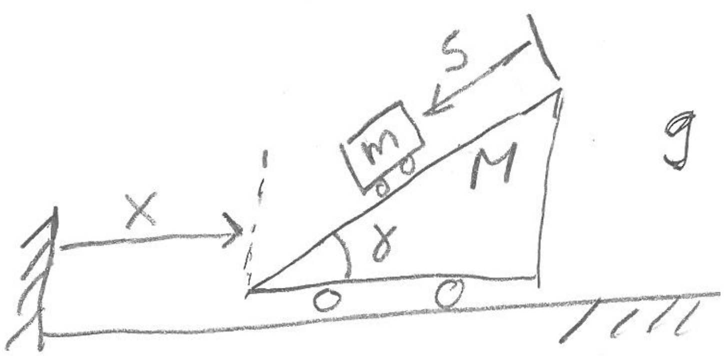
(ii) Some of the statements below are true, some are false, some may be partially true. Say which, and say why (with equations and/or words) clearly enough so that a skeptic would be convinced.

(a) system potential energy is conserved

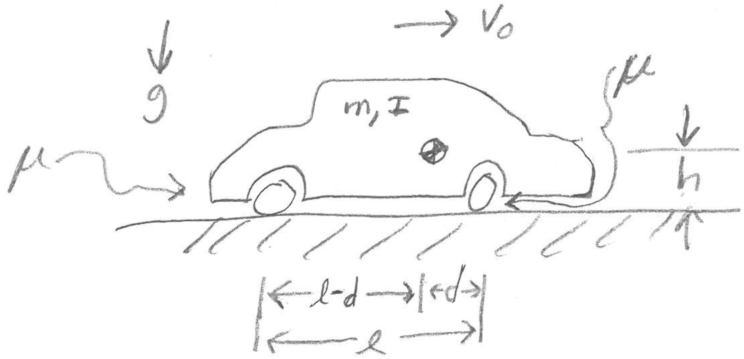
(b) system kinetic energy is conserved

(c) system total energy is conserved

(d) system linear momentum is conserved

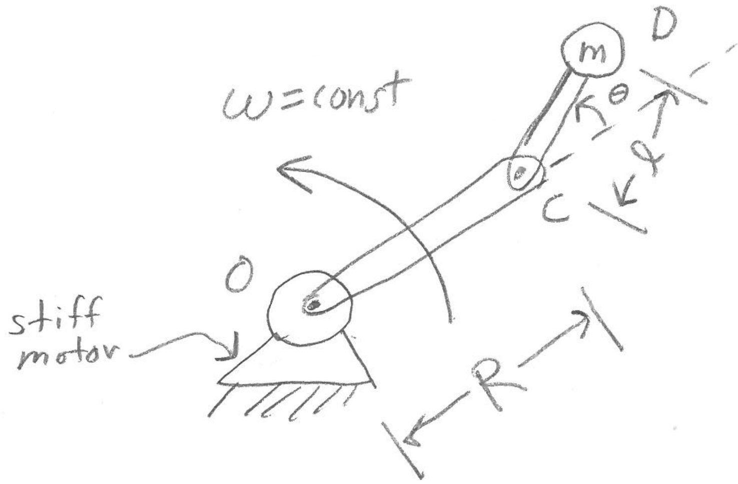


8a) A car with mass m moves at speed v_0 and suddenly slams on the brakes. All four wheels skid with friction coefficient μ (and friction angle ϕ with $\tan\phi = \mu$). Assume the suspension is rigid. In terms of some or all of $d, \ell, h, \mu, \phi, m, I$ and gravity g find how long it takes for the car to come to a stop.



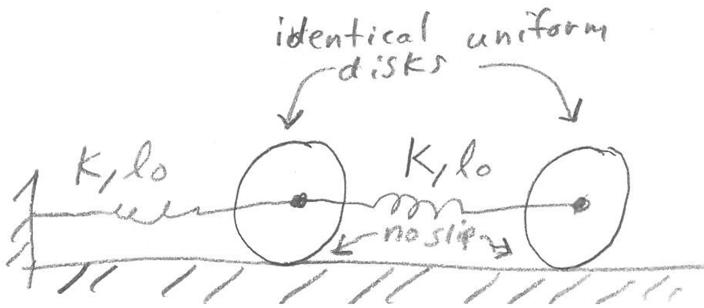
Fall 2012, Final Exam, Problem 8b.

8b) No gravity. One link of a pendulum has radius R and is powered by a stiff motor to rotate at a fixed rate ω. The second link has length ℓ, is massless, and has a point mass m at the end. Find $\ddot{\theta}$ in terms of some or all of θ, $\dot{\theta}$, R, ℓ and ω.



Fall 2012, Final Exam, Problem 9.

9) Two uniform round disks with mass m roll without slipping on a flat plane. They are connected to each other and to the left wall with two springs, both with stiffness k and rest length ℓ_0. Find one normal mode and it's corresponding frequency.



Fall 2012, Final Exam, Problem 10a

10a) This problem concerns the classical 1 DOF spring-mass system with m, k and c.

(a) Make a clear picture of the system.

(b) Using any mechanics method you like, find the equations of motion: $m\ddot{x} + c\dot{x} + kx = 0$.

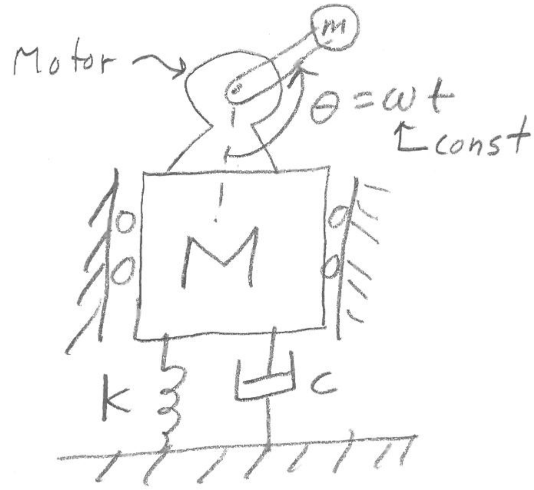
(c) Reduce this to standard form: $\ddot{x} + 2\omega\eta\dot{x} + \omega^2 x = 0$

(d) Define both ω and η with equations. Explain the meaning of both terms with words.

(e) Find an equation for the damped natural frequency ω_d.

Fall 2012, Final Exam, Problem 10b.

10b) A platform and motor with total mass M are supported by a spring (k, ℓ_0) and a dashpot (c). The motor spins at constant rate ω so that $\theta = \omega t$. The motor spins an eccentric mass m which is a distance d from the motor shaft. Find the equations of motion for the mass M consisting of the motor and platform (relative to the static equilibrium position).



Fall 2012, Final Exam, Problem 11.

11) A system has 7 degrees of freedom paramaterized by the components of the 7-element vector $\vec{\boldsymbol{x}}$. The equations of motion, for small motion, are:

$$M\ddot{\vec{\boldsymbol{x}}} + C\dot{\vec{\boldsymbol{x}}} + K\vec{\boldsymbol{x}} = \vec{\boldsymbol{0}}$$

where the matrices M and K are given and symmetric.

(i) Assume that $C = 0$. Write MATLAB commands to find a constant vector $\vec{\boldsymbol{v}}$ and ω so that

$$\vec{\boldsymbol{x}}(t) = \sin(\omega t)\vec{\boldsymbol{v}}$$

is a solution of the governing equations. Just one normal mode solution is desired.

(ii) Clearly define at least one non-zero damping matrix C so that, with an appropriate change of variables the equations can be re-written as a set of decoupled scalar equations of the form:

$$\ddot{r}_i + 2\omega_i \eta_i \dot{r}_i + \omega_i^2 r_i = 0.$$

As for all problems, justify your result.

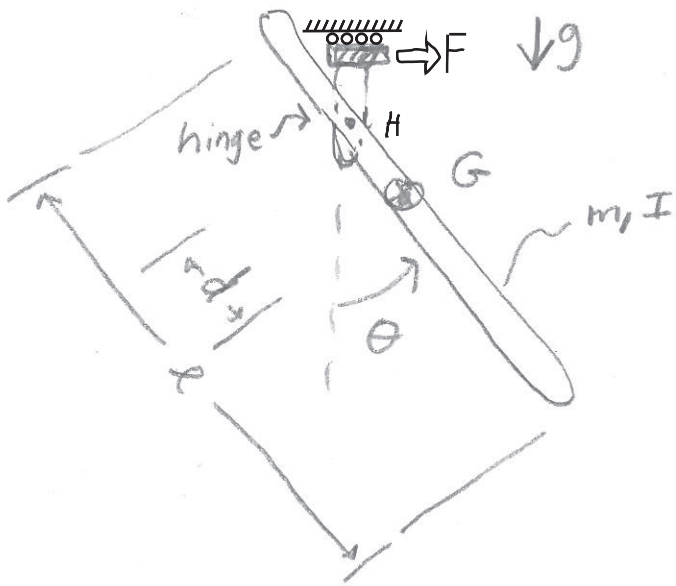
(iii) Clearly define the most general damping matrix C for which, with an appropriate change of variables the equations can be re-written as a set of decoupled scalar equations (as written above).

Fall 2013, Final Exam, Problem 7. (first problem on exam is 7)

7) 2D. A possibly non-uniform stick with length ℓ, mass m and moment of inertia I about its center of mass G is suspended from a hinge on the stick a distance d from G. The hinge is on a massless trolley (with magnetic frictionless wheels) is forced by a force F and has a horizontal acceleration a_H to the right.

(i) Find the equations of motion assuming a_H is known. That is, find $\ddot{\theta}$ in terms of some or all of $m, I, d, \ell, g, a_H, \theta$ and $\dot{\theta}$ (and *not* F).

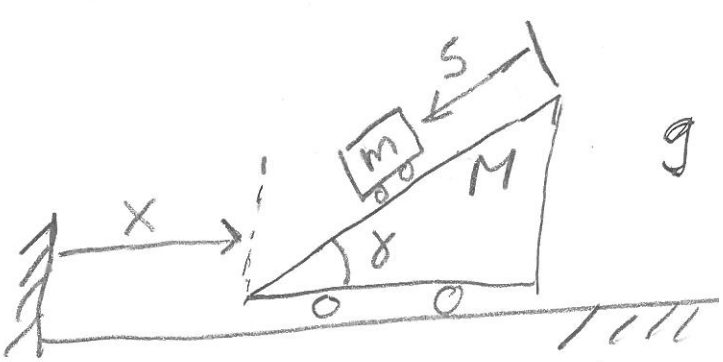
(ii) Assuming $F = 0$, this system has more than one degree of freedom. Assume small θ. Find the normal modes and the angular frequencies of small oscillation in terms of some or all of m, I, d, ℓ and g.



Fall 2013, Final Exam, Problem 8. (first problem on exam is 7)

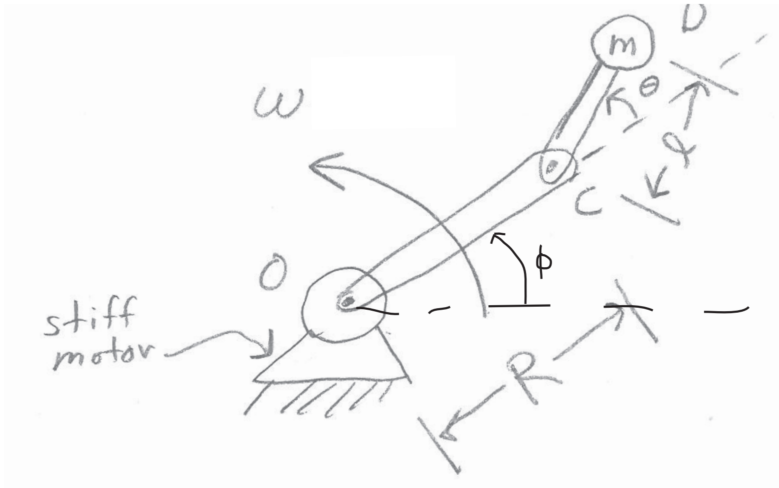
8) A block with mass M slides without friction on a flat level surface. The top surface has slope γ. A smaller block with mass m slides without friction on the sloped top of the lower block. Assume Matlab code has already been written that assigns numerical values to $x, \dot{x}, s, \dot{s}, \gamma, m, M$ and g.

(i) Write Matlab code to find $\ddot{s}$. [*If* you use symbolic commands to generate the equations of motion, you can assume that you have those available as Matlab expressions (That is, I don't expect you to write the commands to convert symbolic expressions into useable matlab). Just, then, clearly explain clearly enough how you would use those expressions. That is, make it clear that you could manage this if a computer was in front of you and you could type 'help' a few times.]



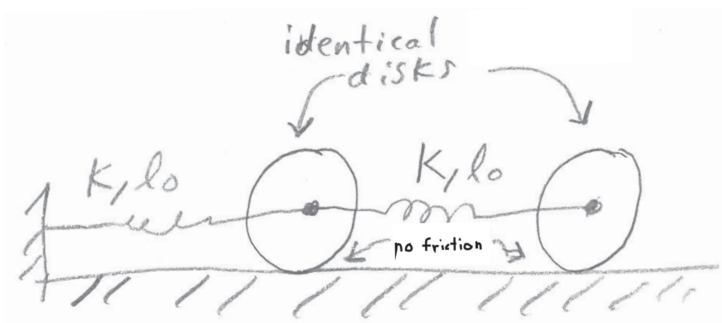
Fall 2013, Final Exam, Problem 9. (first problem on exam is 7)

9) Neglect gravity. One link of a pendulum has radius R and is powered by a strong motor to rotate with given $\phi(t)$ (relative to a fixed horizontal line). Thus $\omega = \dot{\phi}$ and $\alpha = \ddot{\phi}$ are known. The second link has length ℓ, is massless, and has a point mass m at the end. Find $\ddot{\theta}$ in terms of some or all of $\theta, \omega, \alpha, R, \ell, m$ and ω.



Fall 2013, Final Exam, Problem 10. (first problem on exam is 7)

10) Two round disks with mass m and moment of inertia I (about their centers) slide with no friction on a flat plane. They are connected to each other and to the left wall with two springs, both with stiffness k and rest length ℓ_0. Find one normal mode and it's corresponding frequency.



Fall 2013, Final Exam, Problem 11. (first problem on exam is 7)

11) Consider the classical 1 DOF spring-mass system with m, k and c.

(a) Make a clear picture of the system.

(b) Using any mechanics method you like, find the equations of motion: $m\ddot{x} + c\dot{x} + kx = 0$.

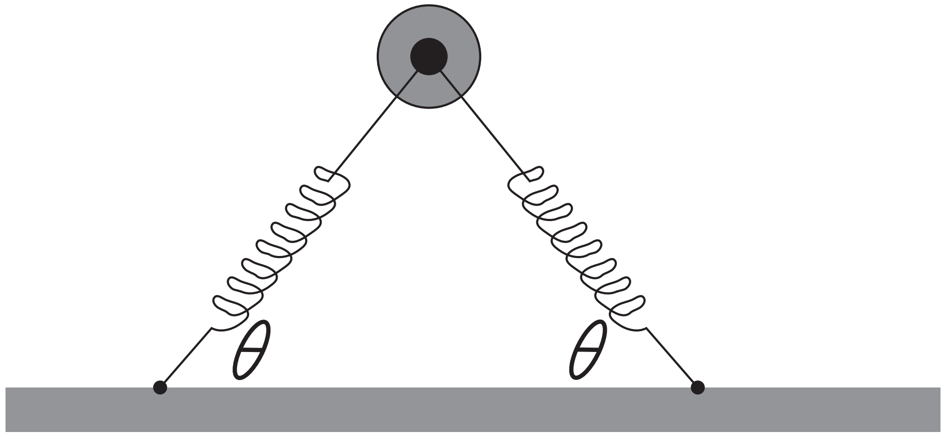
(c) Reduce this to standard form: $\ddot{x} + 2\omega\eta\dot{x} + \omega^2 x = 0$

(d) Define both ω and η with equations. Explain the meaning of both terms with words.

(e) Find the damped natural frequency ω_d in terms of one or both of ω and η.

Fall 2013, Final Exam, Problem 12. (first problem on exam is 7)
12) Neglect gravity. A mass m is supported by two springs (k, ℓ_0) as shown, each making an angle θ from a fixed horizontal line when in the rest position. Assume small (in the usual sense) motions.

(1) Find the normal modes and their angular frequencies in terms of some or all of m, k, ℓ_0 and θ.



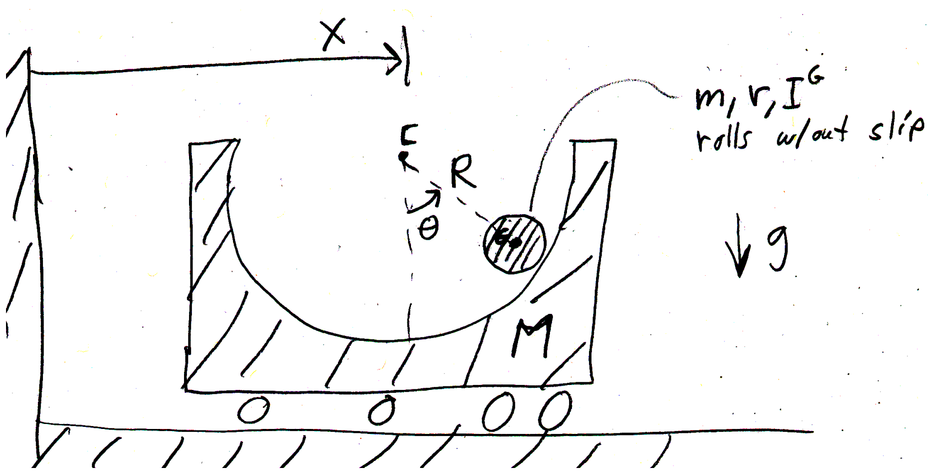
Fall 2014, Final Exam, Problem 7 (first problem on exam is 7)
7) Consider the system below consisting of a cart on a frictionless ground and a disk which rolls, without slip, inside. R is the distance from the center of the hollow cylinder to the center of the rolling cylinder. The radius of the circular trough inside is $R + r$.

a) How many degrees of freedom does this system have?

b) Is energy conserved for the system as a whole?

c) Is linear momentum conserved in the x direction? In the y direction?

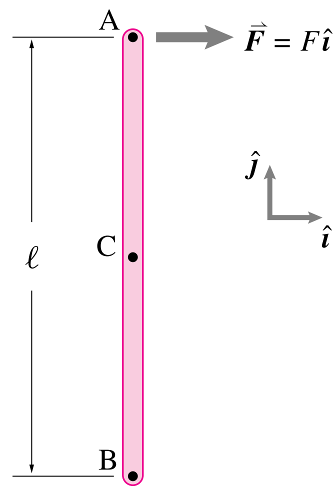
d) Find, by whatever means pleases you (but not allowing computer algebra), one equation of motion of the system below. That is, find one true equation that involves $\ddot{\theta}$ and/or $\ddot{x}$ and some or all of $m, M, r, R, I^G, g, \theta, \dot{\theta}, x$ and $\dot{x}$. You need not find a complete set of ODEs, just an example from the set.



Fall 2014, Final Exam, Problem 8 (first problem on exam is 7)

8) A uniform stick with mass m and length ℓ is initially stationary when the force F is suddenly applied. Various points on the bar have various accelerations in the instant just after force application. What additional force (in addition to $F\hat{\imath}$ at A) has to simultaneously be applied to point C to make, just after force application:

$$\text{the acceleration of point B} = \vec{a}_B = F\hat{\jmath}?$$



Fall 2014, Final Exam, Problem 9 (first problem on exam is 7)

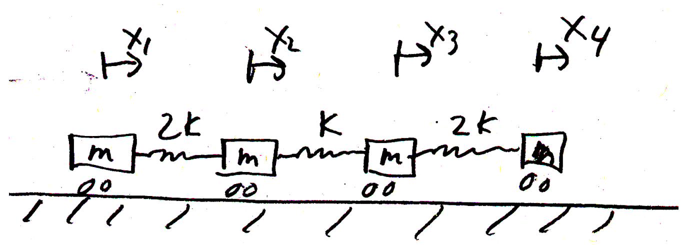
9) 1D. Four equal masses m are in a line. The two outer springs have stiffness $2k$. The middle spring has stiffness k. x_1, x_2, x_3 and x_4 are the displacements relative to a reference equilibrium solution.

a) Find, by any means, as many normal modes and associated frequencies as you can.

b) Given that, in some consistent units, $k = 2, m = 3$ and

$$\vec{x}_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \text{and} \quad \dot{\vec{x}}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

write Matlab commands find the normal modes and use a superposition of them to plot $x_3(t)$ for 5 time units.



Fall 2014, Final Exam, Problem 10 (first problem on exam is 7)

10) Consider the same system as in problem 3: 1D. Four equal masses m are in a line. The two outer springs have stiffness $2k$. The middle spring has stiffness k. x_1, x_2, x_3 and x_4 are the displacements relative to a reference equilibrium solution.

Now consider, additionally, that

i. A horizontal force is applied to mass one: $F_1(t) = F_0 \sin(\omega t)$. Assume ω is not a natural frequency of the unforced system.

ii. Assume you are to add a fifth mass m_5 and spring k_5 to the right of mass 4 and attached only to mass 4.

Questions:

a) Explain, as carefully as you can (carefully enough so we are convinced that you could find a solution in finite time) how to pick k_5 and m_5 so as to make x_{1ss} the steady state motion of mass 1 to be stationary. That is: $x_{1ss}(t) = 0$.

b) Is it possible to solve the problem above for all values of ω that are not natural frequencies. Write a convincing justification of your answer.

c) Given that, in some consistent units, $k = 2, m = 3$ and

$$\vec{\boldsymbol{x}}_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \text{and} \quad \dot{\vec{\boldsymbol{x}}}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

write Matlab commands find the normal modes and use a superposition of them to plot $x_3(t)$ for 5 time units.

Fall 2014, Final Exam, Problem 11 (first problem on exam is 7)

11) Chaplygen Sleigh. 2D. A rigid block slides frictionlessly on the plane but for the skate constraint at C that makes $\vec{\boldsymbol{v}}_C = v\hat{\boldsymbol{\lambda}}$.

Given $x_c, y_c, v, \theta, \dot{\theta}$ and parameters w, d, m and I^G

find $\dot{v}, \dot{x}_c, \dot{y}_c$, and $\ddot{\theta}$.